Mathematical modelling of pile work in sand

Oleg Malyshev

Kyiv National University of Construction and Architecture, geotechnical department

The competition paper for a single pile under load in situ conditions

Introduction

Even in our days it is important correctly determine pile bearing capacity. But it is often very difficult because of the high variety of natural and man-made soil conditions. We propose methodology which based on a mathematic modelling of ground base processions like stress distribution with use variable modulus of deformations for different soil types.

Description of the method

Our calculation methodology based on principles which were proposed by Petrenko G.M. [1]. According to them the curvilinear dependence of pile settlement from the load $S = f(P)$ can be represented as the sum of two curves: the work of pile toe $= f(P_t)$ and the work of the pile on the lateral surface $S = f(P_L)$ (Fig. 1). From the time when compressive force is applied to the pile top dependence between the load and the settlement will be nonlinear. If the load increases to a certain value P_L, the settlement of the pile will be S_L. Further load increase ($P > P_L$) will lead to slipping of the pile in the soils and endless growth of pile settlement. At the same time, the bearing capacity of the pile lateral surface will be constant (P_L).

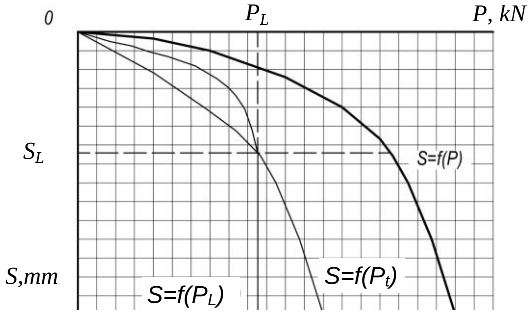

Figure 1. Dividing of the static pile test graph $S = f(P)$ on the graph $S = f(P_L)$ – for the lateral surface and $S = f(P_t)$ – for the toe

When the bearing capacity on the lateral surface will be exhausted an additional load will be received only by the pile toe.

In the limiting state of equilibrium, the shear and normal stresses on the cut surface of the soil shirt relative to the total mass of the soil are in accordance with the Coulomb's law:

$$\tau = \sigma \, tg\varphi + c. \tag{1.1}$$

So elementary lateral surface resistance dP_L of the pile element with a height dz at a depth z will be:

$$dP_L = u(\sigma \, tg\varphi + c) \, dz, \tag{1.2}$$

where u – pile perimeter.

After deepening and during pile work in the soils, on its lateral surface will be act: σ1 – the stress from the soil's weight, σ2 – the horizontal pressure which created by the consolidation of the soil during the pile driving, σ3 – the pressure from the additional soils loading by vertical stresses, arising from the load transmitted by the friction forces on the lateral surface of the pile, that is:

$$\sigma_1 + \sigma_2 + \sigma_3 = \frac{\gamma z(R+r_L)}{(R-r_L)+2\frac{r_б}{\xi}} + \frac{10\mathrm{P}_L(2h-z)z\xi}{\pi h^3 \eta(5r_L+\eta z)}, \tag{1.3}$$

where r_L - reduced radius of the pile lateral surface; γ – soil unit weight; z – distance from the soil surface to the considered point; ξ – the coefficient of the soil lateral pressure; R – outer radius of compacted soil zone; e – coefficient of soil porosity in the natural state; P_L – the load transmitted to the base by friction forces on the pile lateral surface; h – the length of the pile in the soil; η – a tangent of the stress distribution angle in the soil.

Replacing σ in expression (1.2) by expression (1.3) and integrating it at depth z in the range from $z = 0$ to $z = h$, and solving it according to P_L, we obtain the formula for determining the limit pile resistance at the lateral surface:

$$\mathrm{P}_L = \alpha \frac{\pi r_L \left[\frac{\gamma h^2 (R + r_L) tg\varphi}{(R - r_L) + \frac{2r_L}{\xi}} + 2c \cdot h \right]}{1 - \frac{10 r_L \xi \left[h \cdot \eta (3h \cdot \eta + 10 r_L) + 10 r_L (2h \cdot \eta + 5 r_L) \left[ln(r_L) - ln\left(\frac{h \cdot \eta}{5} + r_L \right) \right] \right] \cdot tg\varphi}{h^3 \cdot \eta^4}}. \tag{1.4}$$

Analysis of experimental studies of the basis state tense of friction piles gives us an opportunity to find expression for determining vertical stresses transmitted by the lateral surface in any horizontal plane below the pile toe (fig. 2, a):

$$\sigma_{z\mathrm{L}} = \frac{6\mathrm{P_L}}{\pi(r_\mathrm{L}+k_1(h+z_0))^2} \left[1 - \frac{x}{r_\mathrm{L}+k_1(h+z_0)} \right]^2; \; 0 \le x \le r_L + k_1(h+z_0); \; (z+z_0) > h. \tag{1.5}$$

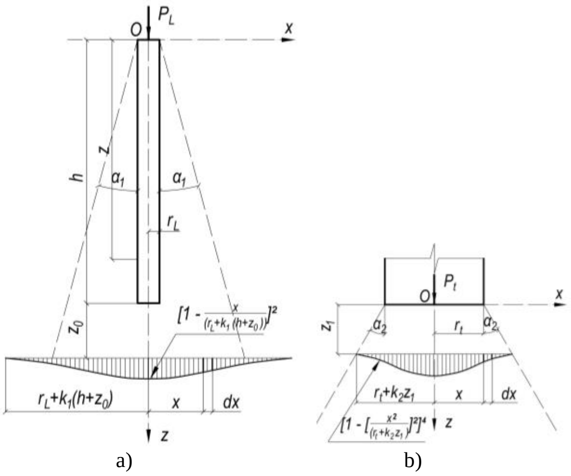

Figure 2. Distribution of vertical stresses in the ground base:
a) at the depth $h + z0$ from the load transferred by pile skin friction forces;
b) at the depth z_1 from the load transferred by the pile toe

Integrating expression (1.5) to z_0 in the range from $z_0 = \delta$ to $z_0 = \infty$, and dividing on deformation module E_L and equating x to 0 after integration we obtain an expression for determining the deformation of the base S_L from the load P_L transferred by pile skin friction forces on ground base:

$$S_L = \frac{6P_L}{\pi E_L k_1} \cdot \frac{1}{(r_L + k_1(h+\delta))}, \qquad (1.6)$$

δ – length of pile toe or the length of compacted soil core under the pile toe.

We can make the same actions with the character of vertical stresses distribution below the pile toe, which appears from the load transferred through it (fig.2, b):

$$\sigma_{zt} = \frac{5P_t}{\pi (r_t + k_2 z_1)^2} \left[1 - \frac{x^2}{(r_t + k_2 z_1)^2}\right]^4; \qquad 0 \le x \le r_t + k_2 z_1. \quad (1.7)$$

And the vertical base deformations S_t with a limited thickness of the ground λ below the pile toe from the load transmitted through the pile toe P_t, after integrating expression (1.7) with z_1 in the range from $z_1 = \delta$ to $z_1 = \lambda$, divide on the module deformation Eв, and equating x = 0 will be:

$$S_в = \frac{5P_t}{\pi E_t} \cdot \frac{(\lambda - \delta)}{(r_t + k_2\delta)(r_t + k_2\lambda)}. \qquad (1.8)$$

The nonlinear relationship between the settlement and the load which transferred by the pile lateral surface and its toe is taken into account by applying variable deformation modules E_L and E_t for the lateral surface and the pile toe [2], which depends on the pressure at the tip of the pile and varies from 1 to 10 MPa. This variable modules of deformation determined for different sand soil type which situated in different parts of country.

When the load on a pile increase by steps using formulas (1.6) and (1.7) it is possible to construct the dependence graph $S = f(P_L)$ and $S = f(P_t)$. The graph $S = f(P)$ is constructed by a graph-analytical sum method of the toe and lateral surface bearing capacity values with the corresponding values of settlement.

Analysis of results

After mathematic modelling of pile work in given ground conditions we were able to build graphics for stress distribution in a base under the pile toe (fig. 3).

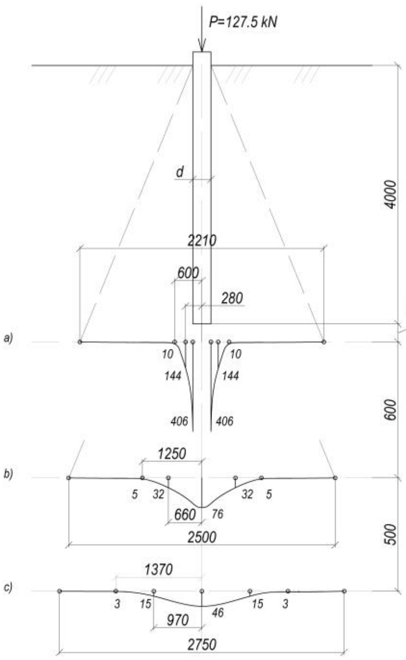

Figure 3. Stress distribution in a ground base from applied on a pile load P=127.5 kN on a depth under pile toe:
a) 400 mm; b) 1000 mm; c) 1500 mm

We can fix stresses extremum on the edges of the pile on a depth of 400 mm. Moving with the depth stresses decrease and spread to a greater horizontally distance. This is corresponding with experimental researches results.

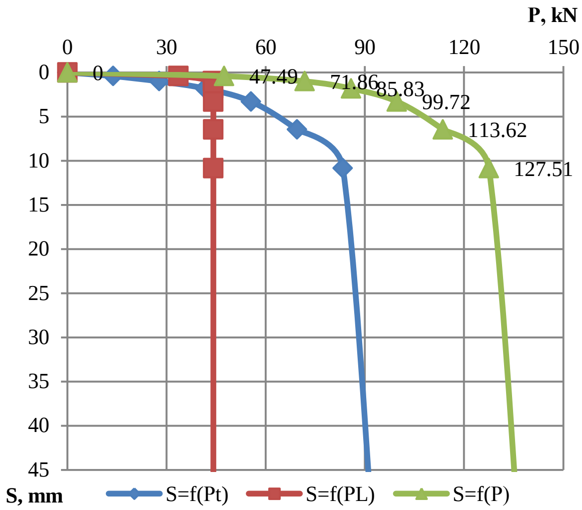

Figure 4. Dependence graph $S = f(P)$ for whole pile;
$S = f(P_L)$ – for the lateral surface; $S = f(P_t)$ – for the toe

Of course according presented methodology we were able to build dependence graph $S = f(P)$ for whole pile, $S = f(P_L)$ – for the lateral surface and $S = f(P_t)$ – for the pile toe.

The limit value of a load on lateral surface is 44 kN which corresponds to the settlement of 1.8 mm. The load on a toe is 83 kN which corresponds to the settlement of 10.87 mm. And ultimate load on a pile is 127.51 kN. After this value we can observe uninterrupted deformation increase with constant load on a pile, which indicates the failure of the pile.

Conclusions

1. Proposed methodology of pile bearing capacity calculation is used formulas with physic-mechanical and deformation characteristics of soils, which increase the accuracy of the calculation.
2. The use of given data can help to construct graphics for pressure in a base under the pile toe which is corresponding with experimental researches and can be used to locate pile in pile foundations more efficiently considering their ground base stress distribution.
2. We have opportunity to build graph $S=f(P)$ and determine bearing capacity of whole pile and its components like a toe or a lateral surface for any value of the settlement.

References

[1] Petrenko G.M. (1968) A new method for calculating piles based on base deformations, *Bases and Foundations: interdepartmental republican scientific collection*. Vol. 1. - pp. 22-30.

[2] *Introduction of new parameters for the calculation of piles by base deformations* (1970) Report on the research work, Kiev, KISI.